Georgia Tech

Jet and Rocket Propulsion

AE4451

LECTURE 9

Sedina Tsikata – January 30, 2023

Overview

what we saw last time:

- chemical equilibrium thermodynamics
  - the equilibriume constant and formation constants
  - combustion example: determining the product state
    - atom balances
    - constraints on mole ratio
    - use of formation constants

today:

- cycle analysis
- performance metrics

Ideal cycles and state diagrams

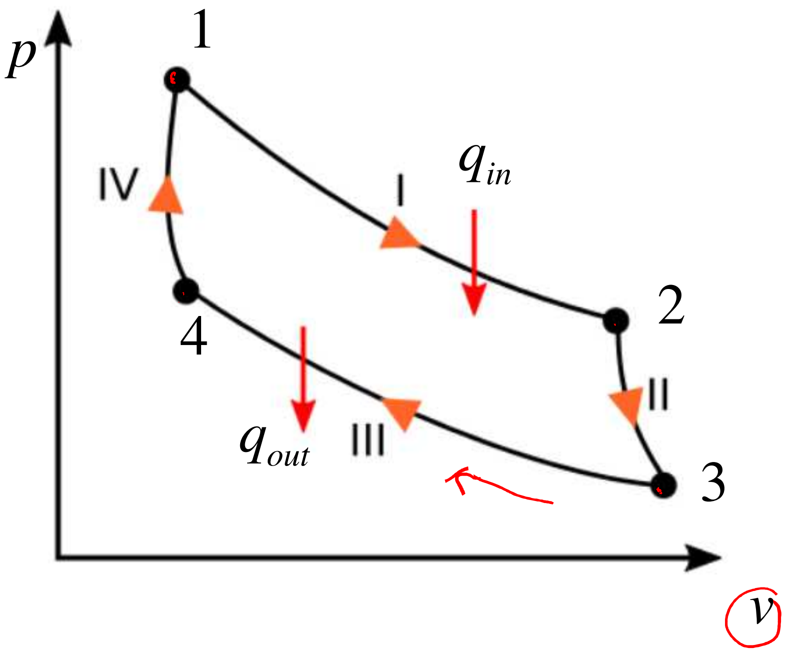
Many real devices will be analyzed with ideal cycle:

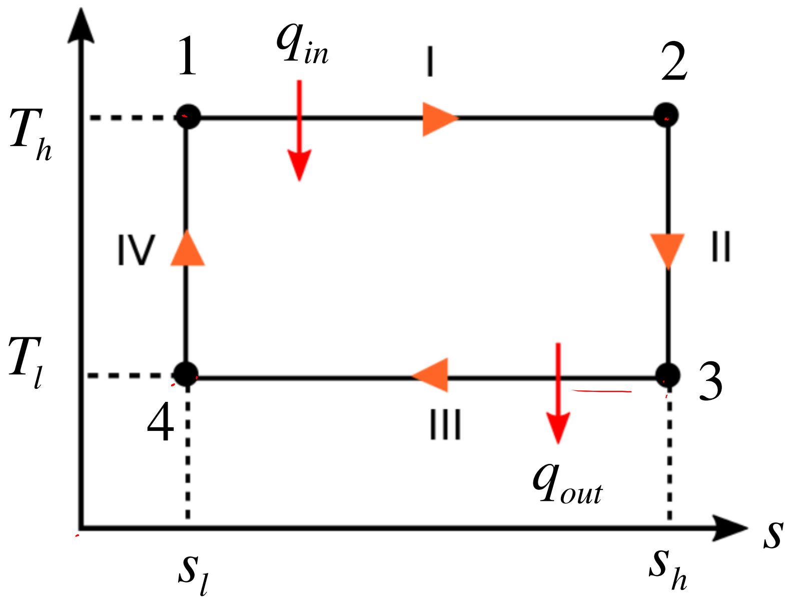
1. assume ideal fluid, e.g., thermally, calorically perfect gas
2. simplify processes, e.g., combustor replaced by non-reacting heat exchanger
3. open system $\rightarrow$ closed system, e.g., interaction with surroundings replaced with heat exchanger
4. assume reversibility, i.e., all components internally reversible

State diagrams are particularly helpful for visualizing cycle processes

Carnot cycle

- The Carnot cycle consists of 4 processes





I. isothermal expansion at T_h

II. adiabatic expansion from T_h to T_l

III. isothermal compression at T_l

IV. adiabatic compression from T_l to T_h

Carnot cycle

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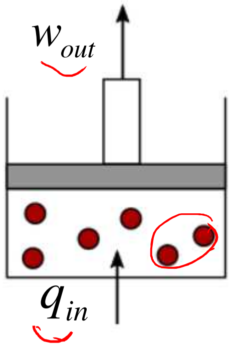
I. isothermal expansion at T_h



- all heat in converted to work

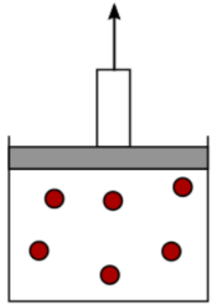
$$w_{out} = -\int_{v_1}^{v_2} pdv = -nRT_h \ln\frac{v_2}{v_1}$$

$$q_{in} = \int_{v_1}^{v_2} pdv = nRT_h \ln\frac{v_2}{v_1}$$

- no change to internal energy and enthalpy

$$\Delta U, \Delta H = 0$$

II. adiabatic expansion from T_h to T_l

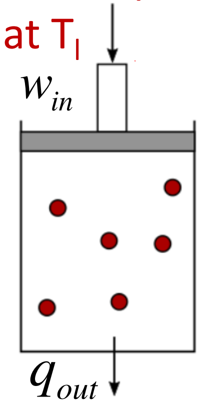


- enthalpy change

$$\Delta H = nc_p dT = nc_p(T_l - T_h)$$

n= number of moles

III. isothermal compression at T_l



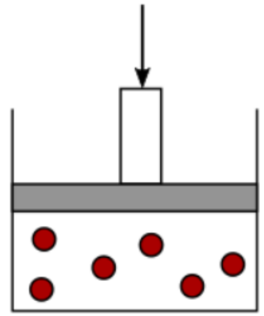
- work done at T_l converted to heat

$$w_{in} = nRT_l \ln\frac{v_4}{v_3}$$

$$q_{out} = -nRT_l \ln\frac{v_4}{v_3}$$

$$\Delta U, \Delta H = 0$$

IV. adiabatic compression from T_l to T_h



- enthalpy change

$$\Delta H = nc_p dT = nc_p(T_h - T_l)$$

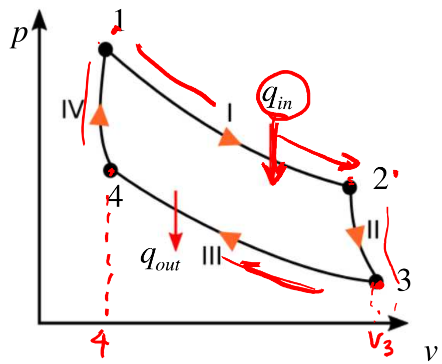
Carnot cycle

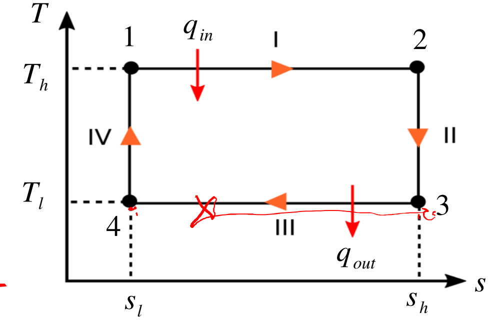
Cycle or thermal efficiency

$$\eta = \frac{\text{net work out}}{\text{energy added as heat}} = \frac{-w_{tot}}{q_{in}} = \frac{\left(nRT_h \ln\frac{v_2}{v_1} + nRT_l \ln\frac{v_4}{v_3} \right)}{nRT_h \ln\frac{v_2}{v_1}}$$

$$T_1 = T_2 \text{ and } T_3 = T_4 \quad \Rightarrow \frac{v_3}{v_4} = \frac{v_2}{v_1}$$

$$= \frac{T_h - T_l}{T_h} \quad \text{so} \quad \boxed{\eta_C = 1 - \frac{T_l}{T_h}}$$



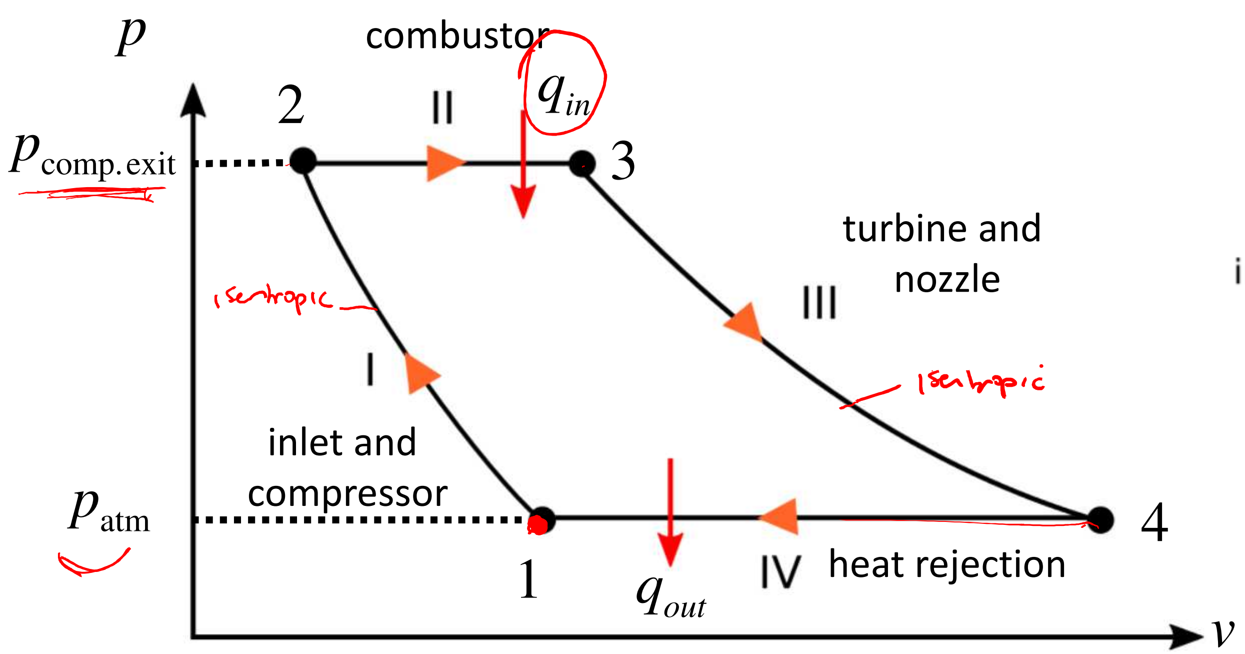


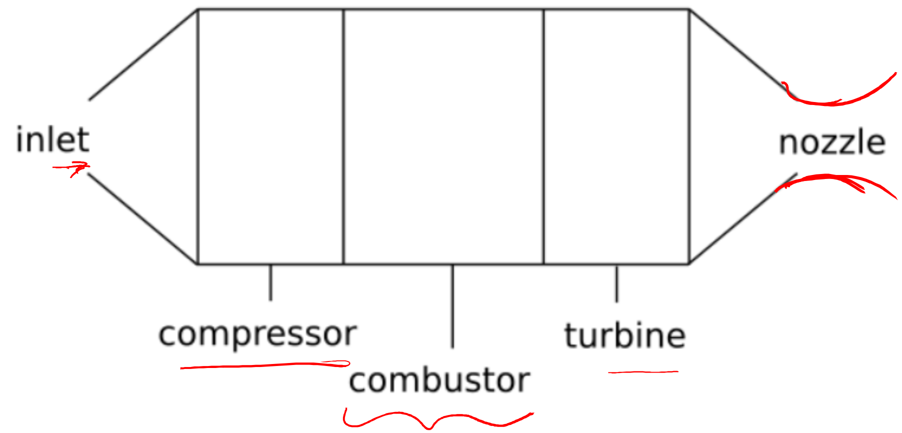
- max η for $T_H \gg T_L$; add heat at as high T as possible (reject heat at as low T as possible)
- even though every process in Carnot cycle is reversible, impossible to get 100% efficiency
- no thermal heat engine has higher η than Carnot

T_L	T_H	η%
300	600	50.0
300	2400	87.5
200	2400	91.7

Brayton cycle

- The Brayton (or Joule) represents gas turbine engine processes



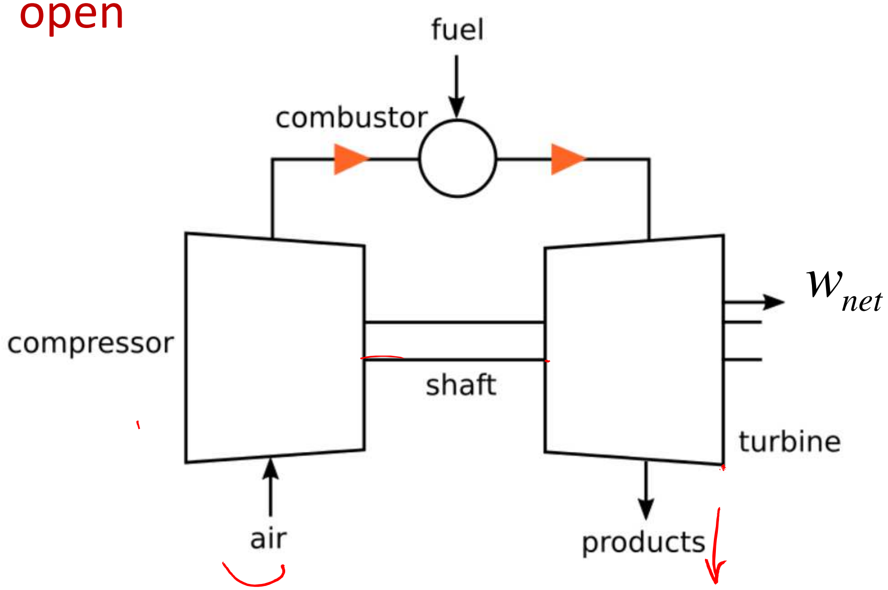


I. adiabatic, reversible compression in the inlet and compressor
II. constant pressure fuel combustion (idealized as constant pressure heat addition)
III. adiabatic, reversible expansion in the turbine and exhaust nozzle; some work out used to drive compressor
IV. cool the air at constant pressure back to its initial condition

Brayton cycle

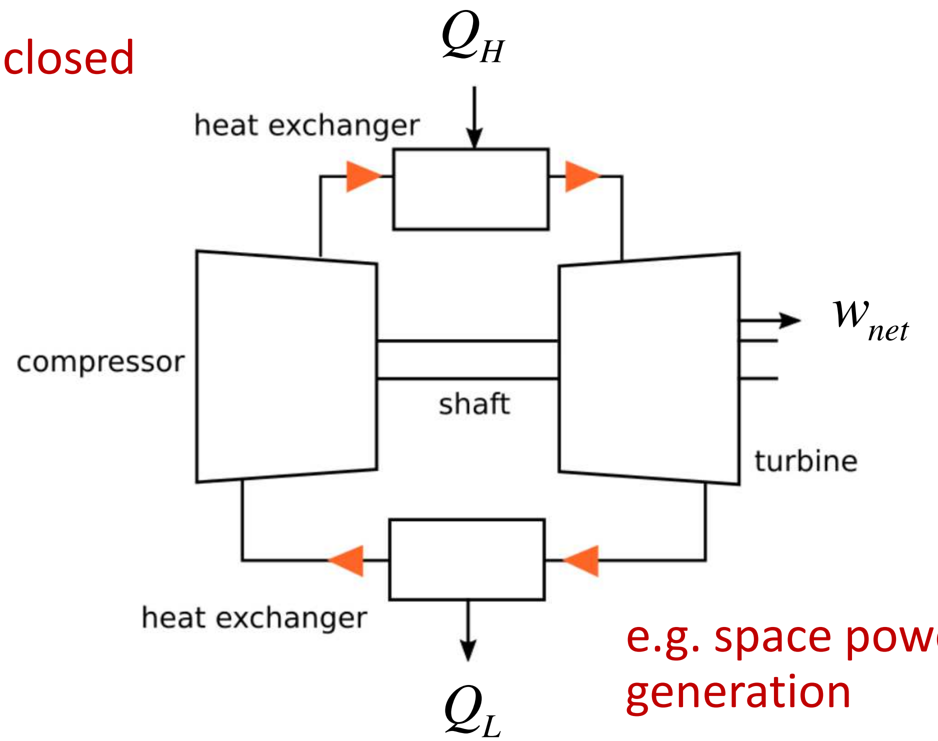
- The Brayton (or Joule) is implemented in closed and open forms

open



e.g. jet propulsion

closed

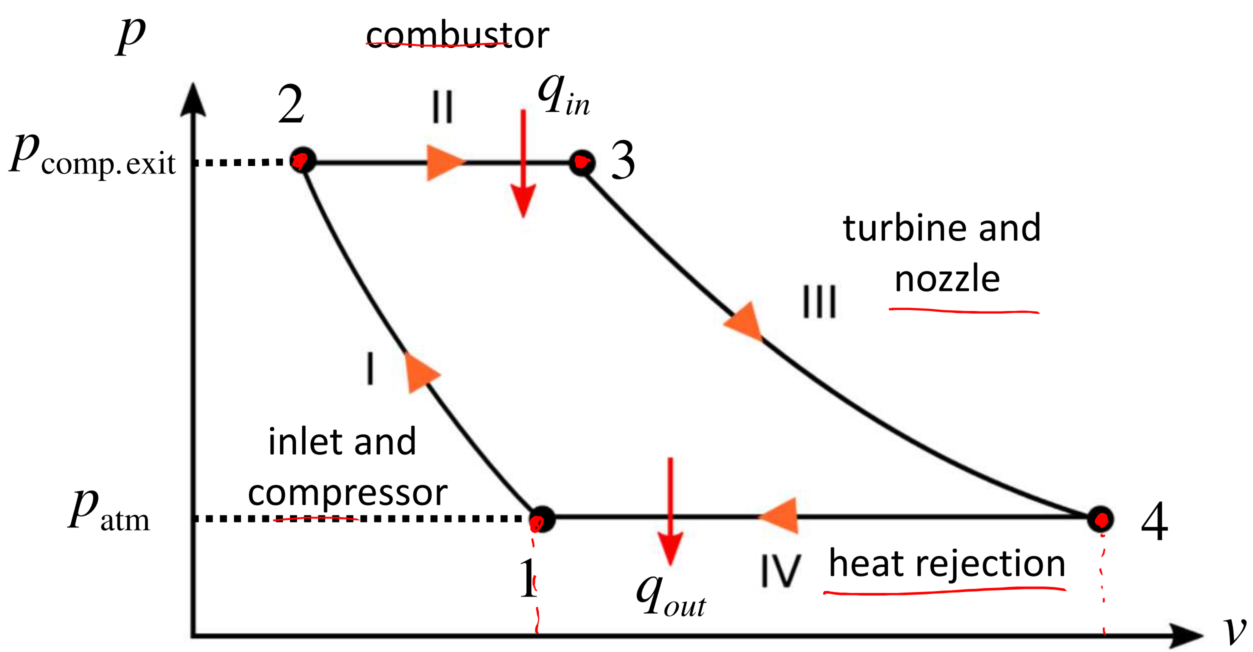


e.g. space power generation

- find:

(i) work done
(ii) heat absorbed
(iii) thermal efficiency of this cycle

Brayton cycle



net work per unit mass

$$w = q_{in} + q_{out} \qquad (q_{out} = \text{negative})$$

constant pressure, reversible

$$dh = c_p dT = dq$$

c_p = const

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

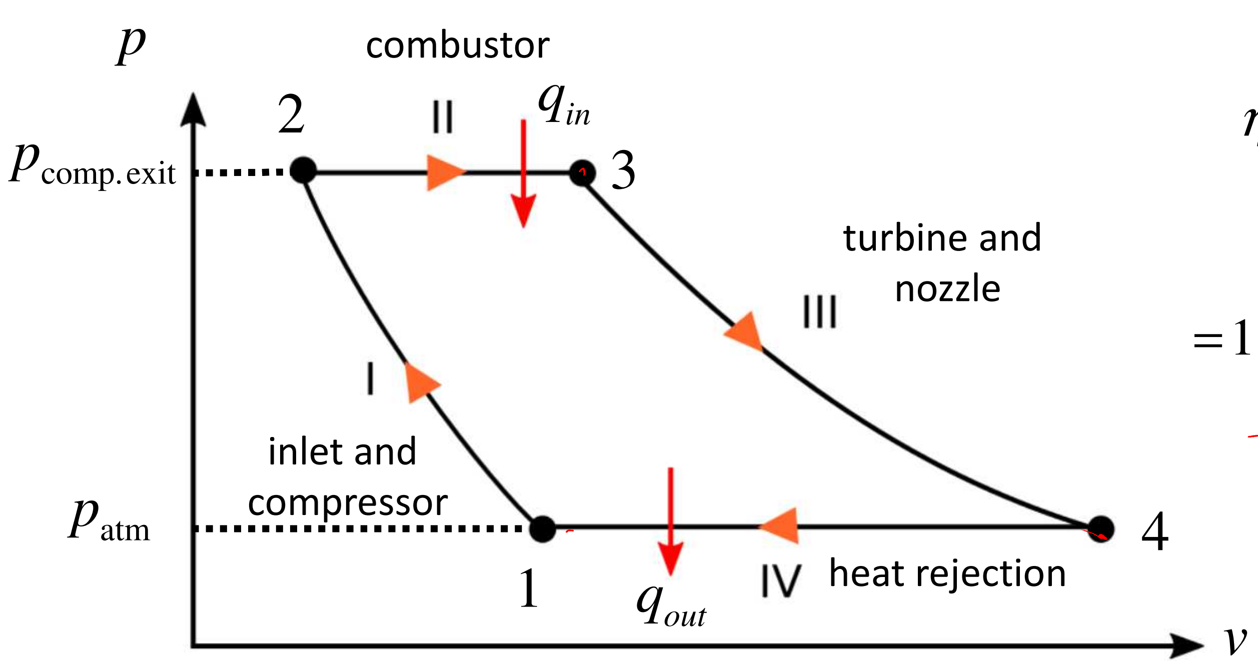
$$q_{out} = h_1 - h_4 = c_p(T_1 - T_4)$$

$$w = c_p(T_3 - T_2) + c_p(T_1 - T_4)$$

$$\eta = \frac{\text{net work out}}{\text{energy added as heat}}$$

$$\eta = \frac{c_p(T_3 - T_2) - c_p(T_4 - T_1)}{c_p(T_3 - T_2)}$$

Brayton cycle



$$\eta = \frac{c_p(T_3 - T_2) - c_p(T_4 - T_1)}{c_p(T_3 - T_2)}$$

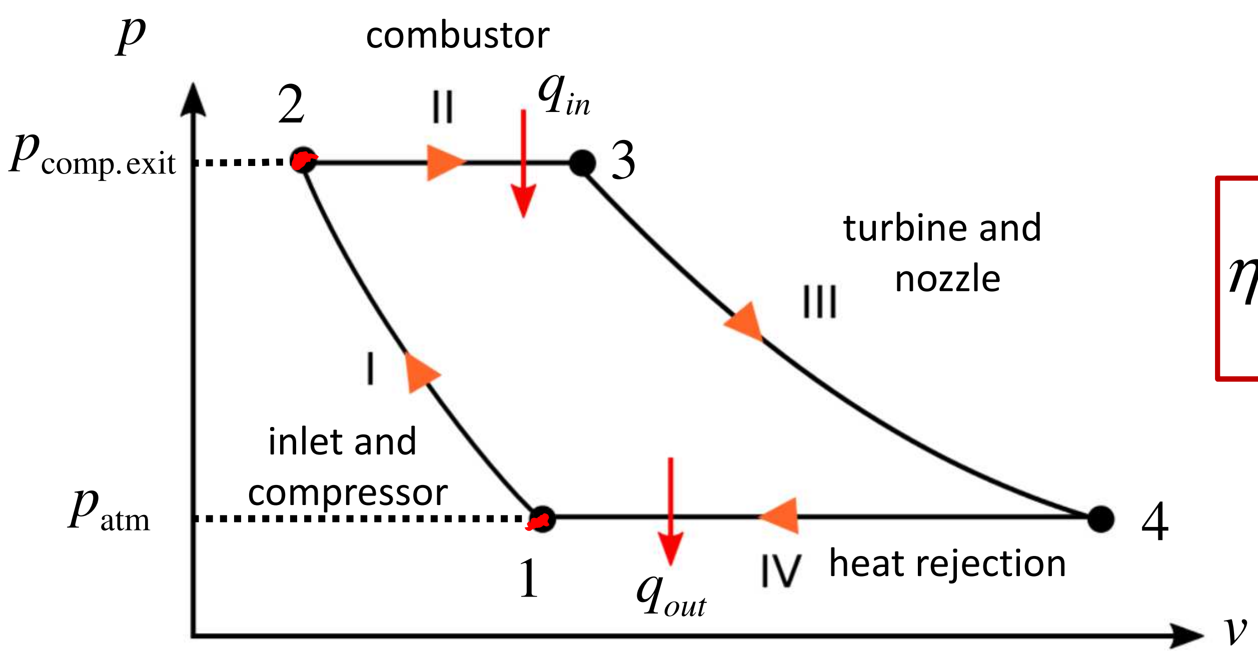
$$= 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$P_1 = P_4 \text{ and } P_2 = P_3$$

$$\frac{P_4}{P_3} = \frac{P_1}{P_2} \Rightarrow \left(\frac{T_4}{T_3}\right)^{\gamma/(\gamma-1)} = \left(\frac{T_1}{T_2}\right)^{\gamma/(\gamma-1)}$$

$$\text{so} \left(\frac{T_4}{T_3}\right) = \left(\frac{T_1}{T_2}\right) \text{and similarly,} \left(\frac{T_4}{T_1}\right) = \left(\frac{T_3}{T_2}\right)$$

Brayton cycle



hence

$$\eta_B = 1 - \frac{T_1}{T_2} = 1 - \frac{T_{\text{atmosphere}}}{T_{\text{compressor exit}}}$$

the efficiency can be written in terms of the temperature ratio across the compressor, called TR
or the pressure ratio PR

$$\eta_B = 1 - \frac{1}{TR} = 1 - \frac{1}{PR^{(\gamma-1)/\gamma}}$$

$$TR = \frac{T_2}{T_1}$$

high TR (or PR) favors higher efficiency, but cannot increase indefinitely

Brayton cycle

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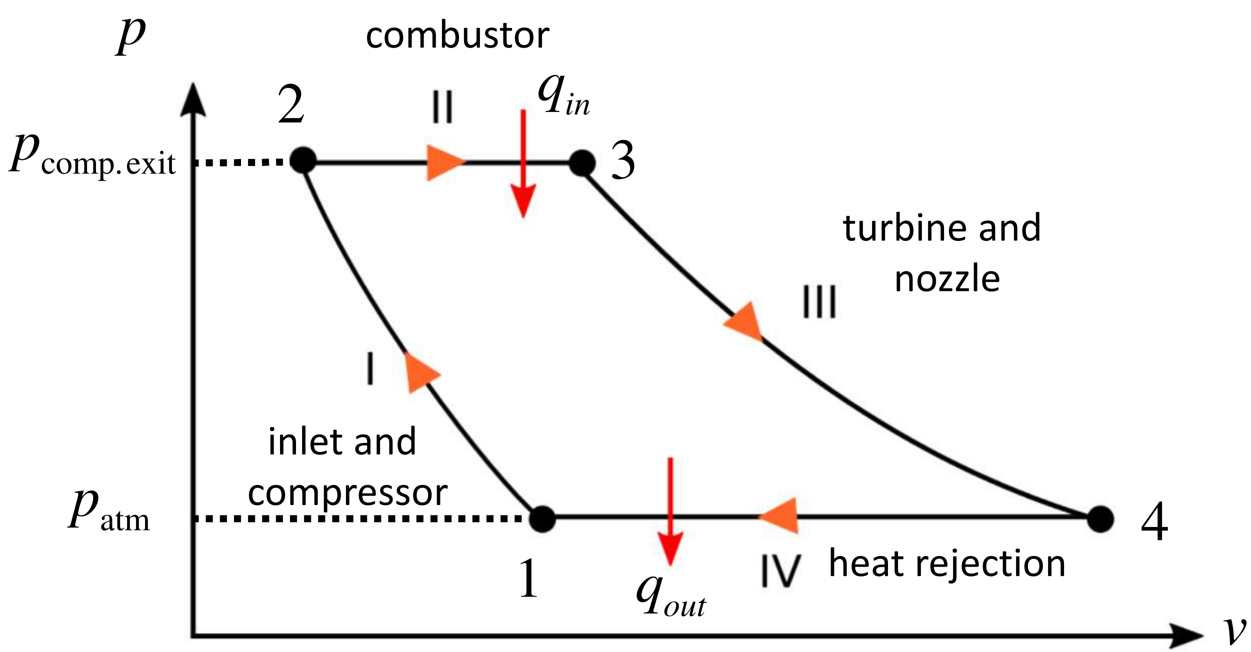

$$\eta_B = 1 - \frac{1}{TR} = 1 - \frac{1}{PR^{(\gamma-1)/\gamma}}$$

η_B maximized if $PR \to \infty$

constraint:

maximum turbine inlet temperature

- set by materials technology and costs
- blade cooling technologies allow increase in turbine inlet temperature

Cycle analysis

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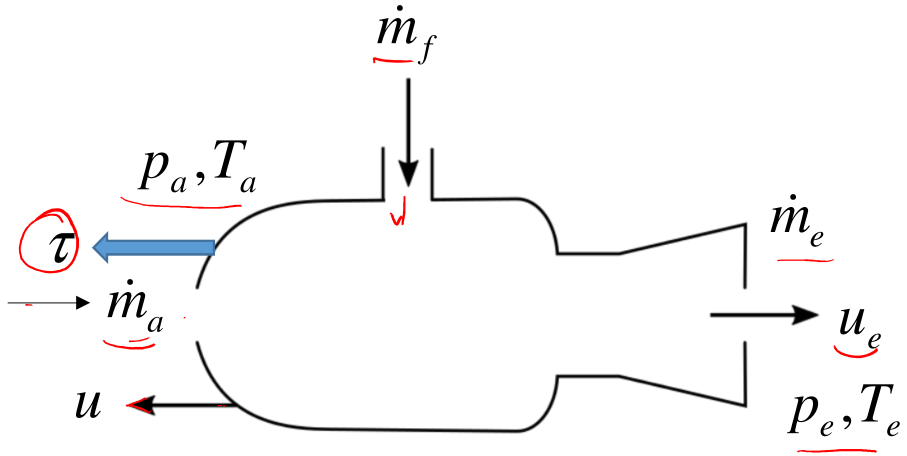
- we will use cycle analysis to predict performance as function of various design variables

- application: air-breathing propulsion systems
  - ramjets, turbojets, turbofans, turboprops

- we need to define performance parameters characterizing such systems
  - specific thrust (ST)
  - specific fuel consumption (SFC)
  - various engine efficiencies, η

Air-breathing system performance

- Jet engine thrust



assumptions:

- steady, uniform, inviscid flow
- single nozzle exhaust stream

from momentum conservation equation,

$$\tau = (\dot{m}_a + \dot{m}_f)u_e - \dot{m}_a u + (p_e - p_a)A_e$$

$$\frac{\dot{m}_f}{\dot{m}_a} = f$$

can be convenient to use fuel to air ratio (*f*) to rewrite this thrust

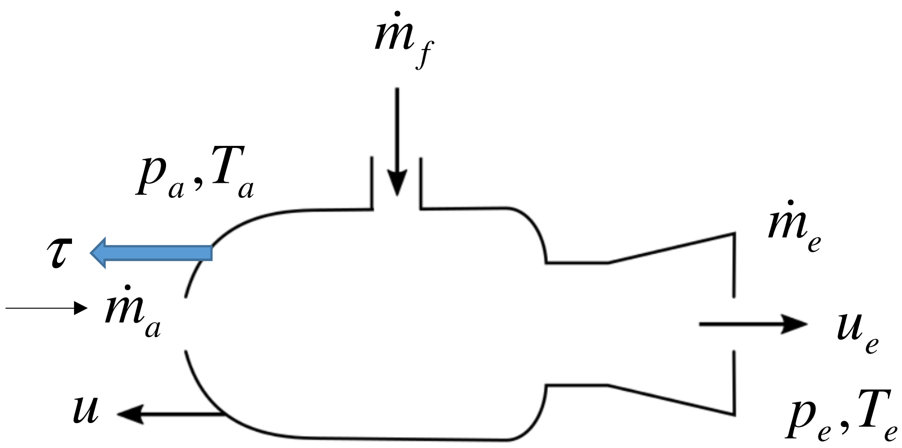
$$\tau = \dot{m}_a[(1+f)u_e - u] + (p_e - p_a)A_e$$

Specific Thrust (ST) $\equiv \dfrac{\tau}{\dot{m}_a} = [(1+f)u_e - u] + \dfrac{(p_e - p_a)A_e}{\dot{m}_a}$

for subsonic nozzle exhausts, $p_e = p_a$

Air-breathing system performance

- Overall efficiency



- can characterize an aircraft propulsion system based on how well it produces the desired output (thrust) given the "cost" input (fuel)

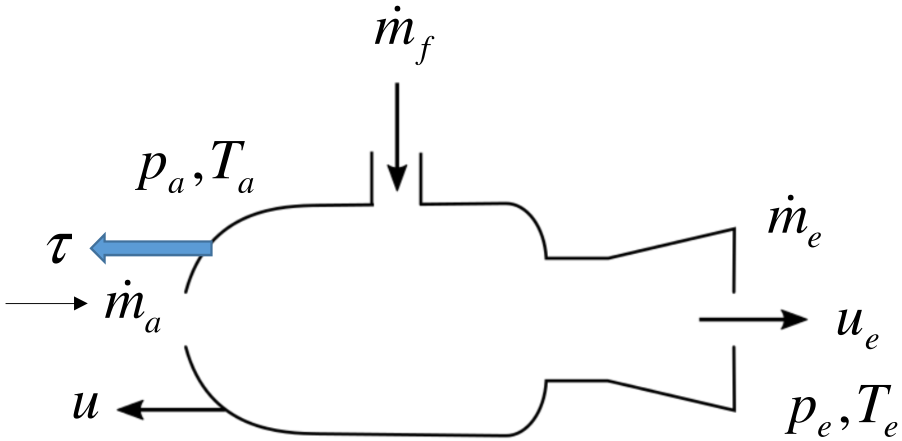
- similar to a cycle efficiency, we can define an overall efficiency for thrust producing engines

$$\text{overall efficiency } \eta_0 = \frac{\tau u}{\dot{m}_f \Delta h_R}$$

i.e. thrust power/heating rate from fuel for thrust producing engines

Air-breathing system performance

- Overall efficiency



- for turboshafts, the goal of the engine is to produce shaft work

here,

$$\text{overall efficiency}\ \eta_0 = \frac{\dot{W}_{shaft}}{\dot{m}_f \Delta h_R}$$

i.e. shaft work/heating rate from fuel

Air-breathing system performance

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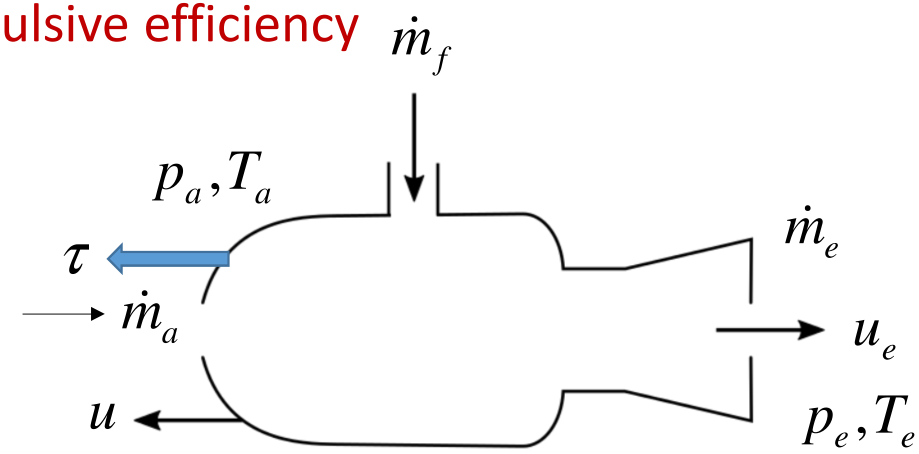
- Thermal and propulsive efficiency



- we can also break down the overall process of how an engine produces thrust into two steps

fuel energy $\rightarrow$ Δkinetic energy of propellant $\rightarrow$ thrust

thermal efficiency   propulsive efficiency

Air-breathing system performance

- Thermal efficiency

\- for thrust produced using nozzles (e.g. turbojet)

thermal efficiency $$\eta_{th} = \frac{\Delta \dot{KE}}{\dot{m}_f \Delta h_R}$$

$$\Delta \dot{KE} = \dot{KE}_{out} - \dot{KE}_{in}$$

$$\Delta \dot{KE} = \frac{1}{2}\left(\dot{m}_a + \dot{m}_f\right)u_e^2 - \frac{1}{2}\dot{m}_a u^2$$

$ke = \frac{1}{2} m v^2$

$\dot{ke} = \frac{1}{2} \dot{m} v^2$

if using fuel to air ratio (*f*) efficiency becomes

$$\eta_{th} = \frac{(1+f)u_e^2 - u^2}{2 f \Delta h_R}$$

Air-breathing system performance

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- Thermal efficiency

- for thrust produced using nozzles (e.g. turbojet), this is just the cycle efficiency for a cycle that outputs kinetic energy (nozzle) instead of work (turbine)

- nozzle exhaust contains gas that is fast (KE) but also hot (thermal energy), so $\eta_{th} < 100\%$

- for a turboshaft engines (and turboprops where most of the output power is to the drive shaft)

$$\eta_{th} \equiv \frac{\dot{W}_{shaft}}{\dot{m}_f \Delta h_R}$$